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A comparative study on Fisher equation using DQM with different forms of B-Spline basis function

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Abstract

When the numerical solution to a set of differential equations is required, regardless of whether the differential equations are ordinary or partial, they are required to be expressed in terms of a basis function. This paper seeks to develop a comparative study of the different types of B-spline basis functions for solving FE using the DQM. The DQM is one of the numerical methods that provides the numerical solution based on the basis functions. FE is a well-known nonlinear reaction-diffusion equation describing the relationship between the diffusion and nonlinear multiplication of a species that has been solved by various numerical approaches. The present work provides a comparison of the numerical solutions obtained using DQM with various forms of B-spline basis functions in terms of CPU time and the errors to investigate the effectiveness of the approach.

Keywords: Differential quadrature method (DQM), Fisher's equation (FEs), SSP-RK43, non-linear differential equation.

1. Introduction

Differential equations are crucial for comprehending scientific and engineering phenomena. Scientists are exploring a range of analytical and numerical methods to address the challenge

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posed by solving the modeled DE. However, it's important to acknowledge that the availability of analytical techniques doesn't always guarantee an exact solution to the DEs. Numerical methods play a pivotal role in achieving the necessary precision when solving DEs.

Due to technological advancements, researchers now have various options for obtaining numerical solutions to DEs through software such as MATLAB, Maple, and Mathematica. They are consistently refining these numerical methods to enhance efficiency and precision in finding solutions. The study presents a novel approach to modifying the existing numerical technique. The study introduces a hybrid approach that employs the cubic B-spline basis function in conjunction with the DQM. This combined technique enhances the precision of solutions for the renowned Fisher's equation (FE) following the discretization of the equation through the collocation method.

In 1937, R.A. Fisher introduced the Fisher equation, a reaction-diffusion equation, to study the spread of advantageous genes within a population due to mutation [1, 2]. Researchers subsequently investigated the application of this equation to various phenomena, including wave propagation and pattern formation, The application of this model extends to the study of cell growth in tissue engineering and the modeling of oscillatory chemical phenomena, to study population dynamics, wound healing, and tumor growth.

The nonlinear partial FRDE is in the form:

$$\frac{\partial w}{\partial t}(x,t) = v\frac{\partial^2 w}{\partial x^2}(x,t) + \rho f(w(x,t)) \tag{1}$$

where, t denotes the time, x represents the spatial coordinate, f is the nonlinear reaction term, v is a constant characterizing the diffusion rate and ρ is the reaction factor. These terms denote the parameters characterizing the phenomenon that is formulated as an FE, Specifically, in the investigation of brain tumors, ' w ' represents the maximum supportable tumor size, the diffusion coefficient v shows the migration of cell and the ρ represents the growth rate.

Similarly, in the context of gene propagation, w signifies the frequency of the mutant gene and ρ The strength of selection favoring the mutant gene is represented by the reaction factor.

In many of the modeled scientific and biological phenomenon, the reaction term

$f(w) = \rho w(1-w)$ and the equation is solved with given initial and boundary constraints:

$$w(x,0) = w_0(x) \ \in [0,1], x \in [-\infty, \infty], \tag{1.1}$$

$$\lim_{x\to-\infty} w(x,t) = 1,\ \lim_{x\to\infty} w(x,t) = 0 \tag{1.2}$$

$$\lim_{x\to\pm\infty} w(x,t) = 0 \tag{1.3}$$

On the availability of the nonlocal conditions (1.1) -(1.2) or the local conditions (1.1) -(1.3) with a finite domain $[x_L, x_R]$, the equation can be solved.

Researchers have extensively investigated both analytical and numerical methods to determine accurate solutions to the equation. One example of solving FE involves employing the space derivative approach with Fourier series [2]. Explicit solutions for FE have been obtained through the application of [3]. The explicit finite difference method has also been applied to solve this equation [4]. Researchers utilize the Petrov-Galerkin finite element method to acquire the numerical solution of the equation [5]. A new class of FDS for solving the FE was constructed by Mickens [6]. The numerical study of the equation was performed [7] using the moving mesh method. A comparative analysis [8] of FDS and the nodal integral method was undertaken, with the FE numerical solution serving as a reference. The solution to the equation, as presented in [9], is obtained using the Adomian decomposition method. A spectral approach using Chebyshev-Lobatto points is developed [10] to solve the equation. A hybrid approach, combining a DQM with a finite difference scheme, has been utilized to solve the FE [11]. In recent years, an array of alternative numerical techniques, including the Quartic B-spline Galerkin method, Tension spline method, Wavelet Galerkin method, boundary integral equation method, and another instance of the DQM, have been effectively employed to solve the FE [12-16].

Researchers have employed a wide range of computational methods to seek the FRDE (Fisher's reaction-diffusion equation) solution numerically. Some of these approaches include the Petrov-Galerkin FEM [17], a pseudo-spectral technique [18, 19], both implicit and explicit FD algorithms [20, 21], an optimized FDS [22], the mesh method [23], the sinc collocation method [24], the B-spline collocation method [25], a MCBS collocation approach [26], the B-spline Galerkin method [27], and the ADM [28, 29]. Additionally, the authors [30] have conducted a comparative analysis of the discrete singular convolution method and for solving the FRDE other three numerical schemes are discussed.

The DQM, introduced by Bellman et al. [31], is an effective technique to solve PDEs. In recent times, the DQM has gained widespread popularity in determining the weighting coefficients. It is applied across numerous articles, utilizing diverse test functions, see [32, 33, 34, 35], MCBDQM [36], cubic B-spline method [37], Hyperbolic B-spline based DQM [38], Quadrature Technique [39] and Crank-Nicolson differential quadrature method (CNDQM) [40].

2. DQM Approach:

The credit for the development of the well-known numerical approach, DQM was given to Bellman et al. [31]. This method has proven to be an effective approach for solving PDEs. By representing derivatives as linear combinations of the function and corresponding weighting coefficients, the method transforms the PDEs into a system of ODEs, facilitating efficient solution. Researchers have used this numerical approach to solve the differential equations arising in the field of science, mathematics and biology [13]-[16].

Let the finite spatial domain of the differential equation is given as [a, b]. It can be discretized in to no. of known points as $a = x_0 < x_1 < \cdots\cdots < x_{n-1} < x_n = b$

The r^{th}derivative of the function can be written as:

$$\left\lfloor \frac{d^r \mathcal{W}}{dx^r} \right\rfloor_{x_i} = \sum_{j=1}^{M} p_{ij}^{(r)} \mathcal{W}(x_j), i = 1\ to\ S, r = 1\ to\ s-1 \quad \ldots.(2.1)$$

where, $p_{ij}^{(r)}$ represents the coefficients to be calculated using the appropriate basis function.

There are a lot of basis functions that can be implemented to approximate the derivative such as the Lagrange's polynomial, radial basis function and simple polynomial etc. In the recent work cubic B-spline basis function in modified form is given as follows:

$$\varphi_i(x) = \frac{1}{h^3} \begin{cases} (x_{i+2} - x)^3, x \in [x_{i+1}, x_{i+2}) \\ h^3 + 3h^2(x_{i+1} - x) + 3h(x_{i+1} - x)^2 - 3(x_{i+1} - x)^3, x \in [x_i, x_{i+1}) \\ h^3 + 3h^2(x - x_i) + 3h(x - x_{i-1})^2 - 3(x - x_{i-1})^3, x \in [x_{i-1}, x_i) \\ (x - x_{i-2})^3, x \in [x_{i-1}, x_i) \\ 0, otherwise \end{cases} \quad \ldots.(2.2)$$

The basis functions are adjusted to transform them into a system of equations with diagonal dominance. The modification in the cubic B-spline basis functions is done at the first two and the last two knot points to deal with the contribution of the spline at the extra know points.

Thus, the value of the function at the knots is defined expressed as:

$$\phi_1(x) = \varphi_1(x) + 2\varphi_0(x)$$
$$\phi_2(x) = \varphi_2(x) - \varphi_0(x)$$
$$\phi_j(x) = \varphi_j(x), j = 3,4, \dots\dots, S-2 \quad \dots.(2.3)$$
$$\phi_{S-1}(x) = \varphi_{S-1}(x) - \varphi_{S+1}(x)$$
$$\phi_S(x) = \varphi_S(x) + 2\varphi_{S+1}(x)$$

To calculate the first-order derivative approximation here using the DQM method utilizing modified cubic basis functions, the first derivative can be written for the grid points $x_i, i = 1,2, \dots\dots\dots, S$ as

$$\phi_k'(x_i) = \sum_{j=1}^{S} p_{ij}^{(1)} \phi_k(x_j), k = 1,2, \dots\dots, S \quad \dots.(2.4)$$

Where, unknown weighting coefficients are in the form of $p_{ij}^{(1)}$ to acquire $\phi_k(x_j)$ which are the modified cubic basis functions. Using the values of $\phi_k'(x_i)'s$ and $\phi_k'(x_i)'s$, for the first grid point taking $i = 1$ the following equations system is obtained

$$X\vec{p}[i] = \vec{Y}[i], i = 1,2, \dots\dots, S$$

Where X is the coefficients matrix given as follow's:

$$\begin{bmatrix} 6 & 1 & & & & & \\ 0 & 4 & 1 & & & & \\ & 1 & 4 & 1 & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & 1 & 4 & 1 & \\ & & & & 1 & 4 & 0 \\ & & & & & 1 & 6 \end{bmatrix},$$

with the weighting coefficients in the form of column as, $p[i]$=$[p_{i1}, p_{i2}, \dots\dots p_{iS}]^T$ and the coefficient vector $\vec{y}[i] = [y_{i1}, y_{i2}, \dots\dots, y_{iS}]^T$ corresponds to grid point x_i, $i = 1,2, \dots\dots, S$ that can be evaluated as follows:

$$Y[1] = \begin{bmatrix} -\frac{6}{h} \\ \frac{6}{h} \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}, Y[2] = \begin{bmatrix} -\frac{3}{h} \\ 0 \\ \frac{3}{h} \\ \vdots \\ \vdots \\ 0 \end{bmatrix}, \ldots\ldots, Y[S-1] = \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ -\frac{3}{h} \\ 0 \\ 3/h \end{bmatrix}, Y[S] = \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \\ -\frac{6}{h} \\ \frac{6}{h} \end{bmatrix}$$

The resulted tri-diagonal equation system provides the weighting coefficient $p_{i1}, p_{i2}, \ldots \ldots p_{iS}\ for\ i = 1,2, \ldots., S$. Substituting the obtained coefficients $p_{ij}^{(1)}$, the coefficients $p_{ij}^{(2)}$for $i = j = 1$ to S evaluated as:

$$p_{ij}^{(2)} = 2p_{ij}^{(1)} \left(p_{ij}^{(1)} - \frac{1}{x_i - x_j} \right) for\ i \neq j \quad \ldots.(2.5)$$

$$p_{ij}^{(2)} = - \sum_{i=1, i\neq j}^{S} p_{ij}^{(2)}$$

3. Numerical example:

Example-1: The FE was presented by Petzold and Ren [41] in study, an investigation into the stability of a moving mesh approach is conducted, focusing on scenarios where the non-linear reactive term (p) exhibits a magnitude substantially greater than the diffusion term, with the reaction rate coefficient (v) having a value greater than or equal to unity. The solution for the above discussed equation is reported in literature [42] as:

$$w(x,t) = \left[1 + exp\left(\sqrt{\frac{p}{6}} x - \frac{5pt}{6} \right) \right]^{-2}$$

Solution of the FE is obtained using the different schemes considering a finite domain as $[-0.2, 0.8]$.

Table no.1: A Comparative analysis of the analytical solution, as compared to solutions derived through various methods, including exact solutions for $t = 0.001$ at $p = 10000$.

x	Exact	B-spline method [25]	Cubic B-spline method [26]	Crank Nicolson DQM method	Hyperbolic B-spline based DQM

				[40].	[38]
-0.2	1.00000	1.00000	1.00000	1.00000	1.000000
-0.1	1.00000	0.99999	0.99999	1.00000	0.999994
0.1	0.9691	0.97203	0.97199	0.9668	0.969136
0.2	0.2578	0.28644	0.29002	0.2398	0.257734
0.3	2.73E-04	0.00032	0.00035	2.37E-04	0.000272
0.4	7.50E-08	0.00000	0.00000	8.82E-08	7.47E-08
0.5	1.99E-11	0.00000	0.00000	2.22E-09	1.99E-11
0.6	5.29E-15	0.00000	0.00000	1.24E-10	5.27E-15
0.7	1.40E-18	0.00000	0.00000	4.63E-12	1.40E-18

Table no.2: A Comparative analysis of the analytical solution, as compared to solutions derived through various methods, including exact solutions for $t = 0.002$ at $p = 10{,}000$.

x	Exact	B-spline method [25]	Cubic B-spline method [26]	Crank Nicolson DQM method [40].	Hyperbolic B-spline based DQM [38]
-0.2	1.00000	1.00000	1.00000	1.00000	1.00000
-0.1	1.00000	1.00000	1.00000	1.00000	1.00000
0.1	0.99992	0.99999	0.99999	0.99992	0.99992
0.2	0.99953	0.99958	0.99959	0.99950	0.99953
0.3	0.97200	0.99454	0.97551	0.97026	0.97195
0.4	0.28375	0.30845	0.32607	0.26831	0.28235
0.5	0.00033	0.00036	0.00045	0.00063	0.00033
0.6	9.16E-08	1.03E-07	0.0000	1.02 E-07	9.08E-08
0.7	2.43E-11	2.93E-11	0.0000	2.67E-10	2.41E-11

Example-2: In this example we have solved the FE

$$\frac{\partial w(x,t)}{\partial t} = v\frac{\partial^2 w}{\partial x^2}(x,t) - qu^2 + pu$$

for t $\in [0,T], 0 < T \leq \infty, -\infty < x < \infty$ with boundary constraints:

$$\lim_{x\to-\infty} w(x,t) = 0.5,\ \lim_{x\to-\infty} w(x,t) = 0$$

And initial constraints:

$$w(x,0) = -\frac{1}{4}\frac{p}{q}\left[\sec h^2\left(-\sqrt{\frac{a}{24r}}x\right) - 2tanh\left(-\sqrt{\frac{a}{24r}}x\right) - 2\right]$$

Exact solution:

$$w(x,t) = -\frac{1}{4}\frac{p}{q}\left[\sec h^2\left(\pm\sqrt{\frac{p}{24r}}x + \frac{5p}{12}t\right) - 2tanh\left(\pm\sqrt{\frac{p}{24r}}x + \frac{5p}{12}t\right) - 2\right]$$

In summary, this equation describes how the density of label particles changes over time. This change is influenced by two factors: the infection rate represented by the term pu- $qu^2$and the diffusion in the surrounding area. The parameter 'pu' quantifies the rate of infection, which depends on the product of infected and uninfected particle densities. The second term illustrates the speed at which infected particles diffuse. The resulting solution manifests as a shockwave-like traveling pattern. The wave's amplitude is directly proportional to $'p'$ and inversely proportional to $'q$,' meaning it grows as $'p'$ increases but diminishes as 'q' increases. The wave's span is determined by $\sqrt{\frac{24r}{p}}$, and its propagation rate is given by $\frac{\frac{5}{\sqrt{6}}}{\sqrt{pr}}$.

Table 2.1: A Comparative analysis of the analytical solution, as compared to solutions derived through various methods, including exact solutions for $t = 2$.

x	Exact solution	B-spline method [37]	Multi-scale analysis [43]	Modified cubic B-spline [26]
-20	0.498652	0.498653	0.498681	0.498652
-16	0.495740	0.495745	0.495130	0.495741
-12	0.486669	0.486679	0.486758	0.486670

-8	0.459478	0.459478	0.459576	0.459477
-4	0.386791	0.386742	0.386681	0.386787
2	0.158850	0.159011	0.158878	0.158859
6	0.041851	0.041877	0.041822	0.041852
10	0.006465	0.006426	0.006455	0.006462
14	0.000755	0.000746	0.000750	0.000754
18	7.92E-05	7.79E-05	7.617E-05	0.000079

4. Conclusion:

Over the years, different numerical methods have been discussed to study the FEs. This review study presents an insight on different methods. In this study, we aimed to showcase recent advancements through two distinct examples and arrived at the conclusion that Hyperbolic B-spline-based DQM and MCBSM are superior to other methods in their respective applications. These approaches also demonstrate their versatility in effectively solving various higher-order partial differential equations, consistently yielding accurate results.

Conflict of Interest:

The authors declare that they have no competing interests.

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